Measures of Dispersion.

In statistic, the measures of dispersion help to interpret the variability of data i.e to know how much homogenous or heterogenous the data is. In simple terms its shows how squeezed Scattered the variable is.

Types:-

There are two types of dispersion method in statistics which are:-

1) Absolute Measures of dispersion
2) Relative Measures of dispersion

(I) Absolute dispersion.

- Range
- Variance
- Standard Deviation
- Quartiles Deviation
- Mean Deviation.

(II) Relative Measures of dispersion.

- Co-efficient of Range
- Co.eff. of Variance
- Co-eff. of Standard Devi
- Co-eff. of Quartile Dev.
- Co-eff. of Mean Dev.

Properties of Good Measures of dispersion:-

(1) It should be simple to understand.

(2) It must be easy to calculate

(3) It must be based on all the ~~ites~~ items of the series.

(4) It should not be unduly affected by extreme items

(5) It should be least affected by the fluctuations in sampling.

(6) It should be capable of further statistical treatment

Range :– Range is the simplest measures of dispersion which is determined by two extreme values of the observations and it is the difference between largest and the smallest value in a distribution.

Merits :–

(1) Simple to understand and easy to calculate

(2) It present a broad picture of the data

Demerits :–

(1) Gets affected by the extreme items,

(2) Does not take into consideration towards most of ~~them~~ the items and their deviation.

(3) Doesn't give ~~suspen~~ reasonable picture of the data.

(4) It is influenced by fluctuations of sampling.

It is calculated by the following formula.

$$\boxed{Range(R) = L - S}$$

where,

R = Range,

L = Largest value

S = Smallest value.

& Co-efficient of Range.

$$\boxed{Co.eff.(R) = \frac{L-S}{L+S}}$$

Example :– Calculate the range & its co-efficient ~~of~~ from the data given.

100 120 160 200 165 180, 140 150

soln R = L − S

L = 200 and S = 100

$R = L - S$

$200 - 100$

$R = 100$

Co-efficient of Range $= \frac{L-S}{L+S} = \frac{200-100}{200+100} = \frac{100}{300}$

$= .33$

For Discrete Series :-

Example :- Calculate the range & its co-efficient from the following data :-

Marks	25	30	35	60	75
No. of students	4	8	10	6	3

Soln

$R = L - S$

$= 75 - 25 = 50$

Co-efficient $= \frac{L-S}{L+S} = \frac{75-25}{75+25} = \frac{50}{100} = 0.5$

For Continuous Series :-

Example :- Find out Range and its co-efficient from the following data

Wages	50-60	60-70	70-80	80-90	90-100
No of workers	7	9	18	13	19

Soln.

$R = L - S$

$L = 100 \quad S = 50$

$R = L - S = 100 - 50 = 50$

Coefficient $= \frac{L-S}{L+S} = \frac{100-50}{100+50} = \frac{50}{150} = 0.33$

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Example:- Find out the largest value with the help of following information.

Smallest value = 20

Co-efficient = .5

$$\text{Co-efficient of Range} = \frac{L-S}{L+S}$$

$$.5 = \frac{L-20}{L+20}$$

$$.5L + 10 = L - 20$$

$$.5L - L = -20 - 10$$

$$-.5L = -30$$

$$.5L = 30$$

$$L = \frac{30 \times 10}{5} = \frac{300}{5} = 60$$

∴ Largest value = 60 Ans

Varience :- Varience is also called second moment of dispersion. This method was devised to avoid the unjustifiable practice of ignoring + and − signs in mean deviation. In this method, the deviations are squared to convert all the −ve values also in +ve it is denoted by σ

$$\sigma = \frac{\Sigma d^2}{N} \text{ or } \frac{\Sigma f d^2}{N}.$$

Mean Deviation :-

Co-efficient of Mean Deviation:-

Co-efficient of Mean Deviation from Mean = $\frac{MD}{\bar{x}}$

" " " " from Median = $\frac{MD}{M}$

" " " " from mode = $\frac{MD}{Z}$

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Calculation of mean deviation in Individual series.

It is calculated by the following formula.

$$MD = \frac{\Sigma|D|}{N}$$

where M.D = Mean Deviation.

|D| = Deviation from mean or median ignoring (+) & (−) signs.

N = No of items.

example :- Calculate Mean deviation and its co-efficient from Mean & Median with the help of following data.

Marks :- 40 50 60 70 90 100 150

Soln Calculation of MD from mean.

Sl NO	Marks	Deviation from 80
1	40	40
2	50	30
3	60	20
4	70	10
5	90	10
6	100	20
7	150	70
N = 7	$\Sigma X = 560$	$\Sigma\|D\| = 200$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{560}{7}$$

$$\bar{X} = 80$$

$$MD = \frac{\Sigma|D|}{N} = \frac{200}{7}$$

$$MD = 28.57$$

$$\text{Co-efficient of MD} = \frac{MD}{\bar{X}} = \frac{28.57}{80} = .357$$

∴ co-efficient = .357 Ans

Calculation of MD from Median:

Sl.No	Marks(x) X	Deviation from 70.
1	40	30
2	50	20
3	60	10
4	70	0
5	90	20
6	100	30
7	150	80
N=7		$\Sigma\|D\| = 190$

$M = \text{Size of } \left\{\frac{N+1}{2}\right\}^{th} \text{ item}$

$= \left\{\frac{7+1}{2}\right\} = \frac{8}{2} = 4$

4th item

Size of 4th item = 70

$MD = \frac{\Sigma|D|}{N} = \frac{190}{7} = 27.14$

$\text{Co-efficient of MD} = \frac{MD}{M} = \frac{27.14}{70} = 0.388$

Calculation of Mean Deviation in Discrete Series.

It is calculated by the following formula.

$$MD = \frac{\Sigma f|D|}{N}$$

where, N = Total of frequency.

Example :- Calculate the mean deviation from median & Mean from the following data

Wage(in Rs) :-	5	10	15	25
No. of workers -	4	6	8	12

Calculation of M.D from Median.

Wage in Rs. x	No of women f	c.f	Deviation $\|D\|=15$	$f\|D\|$
5	4	4	10	40
10	6	10	5	30
15	8	18	0	0
25	12	30	10	120
	N=30			$\Sigma f\|D\|=190$

$M =$ Size of $\left\{\frac{N+1}{2}\right\}^{th}$ item.

$= \left\{\frac{30+1}{2}\right\} = \left\{\frac{31}{2}\right\} = 15.5$

Size of 15.5^{th} item lies in c.f 18 so that
Size of item is 15

$MD = \frac{\Sigma f|D|}{N} = \frac{190}{30} = \frac{19}{3} = 6.4$

co-efficient of $MD = \frac{MD}{M} = \frac{6.4}{15} = 0.4$

From Mean.

x	f	fx	$\|D\|(16.7)$	$f\|D\|$
5	4	20	11.7	46.8
10	6	60	6.7	40.2
15	8	120	1.7	13.6
25	12	300	8.3	99.6
	N=30	$\Sigma fx = 500$		200.2

$\Sigma f|D| = 200.2$

$\bar{X} = \frac{\Sigma fx}{N} = \frac{500}{30}$

$\bar{x} = 16.7$

$MD = \frac{\Sigma f|D|}{N.}$

$MD = \frac{200.2}{30} = 6.68$

Co-efficient $= \frac{MD}{\bar{X}} = \frac{6.68}{16.7} = 0.4$

Calculation of Mean Deviation in continuous Series

Example Calculate the MD from the following data

Marks	0-10	10-20	20-30	30-40	40-50
No of students	5	7	3	8	10

Marks C.I.	No of students f	X MV	fx	28.4 |D|	f|D|
0-10	5	5	25	20.4	102
10-20	7	15	105	13.4	93.8
20-30	3	25	75	3.4	10.2
30-40	8	35	280	6.6	52.8
40-50	10	45	450	16.6	166
	N=33		Σfx 935		$\Sigma f\|D\|$ = 424.8

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{935}{33} = 28.4$$

$$MD = \frac{\Sigma f|D|}{N} = \frac{424.8}{33} = 12.88$$

$$\text{co-efficient} = \frac{MD}{\bar{x}} = \frac{12.88}{28.4} = 0.45$$

Standard Deviation (S.D)

Standard Deviation is the measures of dispersion which is always calculated on the basis of Arithmatic mean. The deviation obtained from Arithmatic mean are squared and its square roots is obtained to give the value of standard deviation. The standard deviation of a set of observations in the square root of average of the squar deviations from mean. It is denoted by "Sigma". When the σ is deviated by the mean of the series we get the co-efficient of standard deviation.

co-efficient of S.D $= \frac{\sigma}{\bar{X}}$

Merits :-

1) Makes use of full information and is based upon all the items of the series.
2) It is defined rigidly as it is always calculated from airthmatic mean.
3) It is easy to understand and simple to calculate.
4) It is based upon ~~[illegible]~~ correct mathematical process.
5) It is provides units of measurment for the normal distribution.

Demerits :-

1) If the data is vast, it involves tedious calculation.
2) Extreme values get more weight because the value of deviation are squared up.

Mathematical Properties

The mathematical properties of S.D are :-

I) Σd^2 or Σfd^2 is the minimum because the deviation are computated on the basis of arithmatic mean.

II) In a Normal distribution $\bar{X} \pm \sigma$ covers 68.27%

of the items $\bar{X} \pm 2\sigma$ covers 95.45% of the items
and $\bar{X} \pm 3\sigma$ covers 99.73% of the items.

iii) If the series is of natural number, standard Deviation can be calculated from the following formula -

$$S.D = \sqrt{\frac{1}{12}(n^2-1)}$$

Where (n) denoted the number of items.

<u>Calculation of Standard Deviation in Individual Series.</u>

There are ~~three~~ two methods to calculate S.D

1) Direct Method

2) Short cut Method

<u>Direct Method</u>

$$\sigma = \sqrt{\frac{\Sigma x^2}{N}}$$

Where,

σ = Standard Deviation

x^2 = square of deviations

N = Number of items

<u>Example</u> :- Calculate Standard Deviation and its Co-efficient from the following data :-

X :- 100 90 120 110 80 70 150 130 50 100

P.T.O

Variable X	Deviation from 100 x	Square of deviations x^2
100	0	0
90	-10	100
120	+20	400
110	+10	100
80	-20	400
70	-30	900
150	+50	2500
130	+30	900
50	-50	2500
100	0	0
$\Sigma X = 1000$	$\Sigma x = 0$	$\Sigma x^2 = 7800$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{1000}{100} = 100$$

$$\bar{X} = 100$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{7800}{10}} = \sqrt{780}$$

$$\sigma = 27.9$$

Short Cut Method :-

i) $\sigma = \frac{1}{N}\sqrt{\Sigma dx^2 . N - (\Sigma dx)^2}$

ii) $\sigma = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma dx}{N}\right)^2}$

iii) $\sigma = \sqrt{\frac{\Sigma dx^2 - n(a - x^2)}{x}}$

Example :- Calculate Standard Deviation from the following data :-

Marks :- 60 70 65 80 85 90 95 100 115, 120

Marks x	Deviation from Assumed Mean (90) dx	dx^2
60	-30	900
70	-20	400
65	-25	625
80	-10	100
85	-5	25
90	0	0
95	+5	25
100	+10	100
115	+25	625
120	+30	900
N=10	$\Sigma dx = -20$	$\Sigma dx^2 = 3700$

$$\sigma = \frac{1}{N}\sqrt{\Sigma dx^2 . N - (\Sigma dx)^2}$$

$$= \frac{1}{10}\sqrt{3700 \times 10 - (-20)^2}$$

$$= \frac{1}{10}\sqrt{37000 - 400}$$

$$= \frac{1}{10}\sqrt{3600}$$

$$= \frac{1}{10} \times 191.3 = 19.13$$

$\sigma = 19.13$ **Ans.**

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Calculation of Standard Deviation in Discrete Series.

It is calculated by two methods :-

(1) Direct Method

(2) Short Cut Method

(1) Direct Method

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N}}$$

N = Total of frequencies

Example :- Calculate Standard Deviation with the help of following data :-

Size	5	15	25	35	45	55	66
Frequency	185	77	34	180	136	23	50

Size	f	fx	Deviation (29)	x^2	fx^2
5	185	925	-24	576	106560
15	77	1155	-14	196	15092
25	34	850	-4	16	544
35	180	6300	+6	36	6480
45	136	6120	+16	256	34816
55	23	1265	+26	676	15548
66	50	3250	+36	1296	64800
	N=685	Σfx = 19865			Σfx^2 243840

$$\bar{X} = \frac{\Sigma fx}{N} = \frac{19865}{685} = 29$$

$$\bar{X} = 29$$

$$\sigma = \sqrt{\frac{\Sigma f x^2}{N}} = \sqrt{\frac{243840}{685}}$$

$$= \sqrt{356} = 18.87$$

$\sigma = 18.87$ Ans.

(2) Short cut Method

It is calculated by the following formula.

$$\sigma = \frac{1}{N}\sqrt{\Sigma f dx^2 \cdot N - (\Sigma f dx)^2}$$

Example :- Calculate the standard Deviation from the following data :-

Daily wage in Rs. :-	2	4	6	8	10
No. of workers	4	6	8	7	5

Soln.

Wage in (Rs) X	No. of workers f	Dev. from (6) dx	fdx	fdx²
2	4	-4	-16	64
4	6	-2	-12	24
6	8	0	0	0
8	7	+2	+14	28
10	5	+4	+20	80
	N = 30	Σdx.	Σfdx = +6	Σfdx² = 196

$$\sigma = \frac{1}{N}\sqrt{\Sigma f dx^2 \cdot N - (\Sigma f dx)^2}$$

$$= \frac{1}{30}\sqrt{196 \times 30 - (6)^2}$$

$$= \frac{1}{30}\sqrt{5880 - 36}$$

$$= \frac{1}{30}\sqrt{5844}$$

$$= \frac{1}{30} \times 76.4 = 2.55 \text{ (Approx.)}$$

$$\boxed{\sigma = 2.55} \text{ (Approx) Ans.}$$

<u>Calculation of Standard Deviation in Continuous Series.</u>

It is calculated by three methods :—

(1) Direct Method
(2) Short cut Method
(3) Step Deviation Method

(1) <u>Direct Method</u> :— It is calculated by the following formula :—

$$\boxed{\sigma = \sqrt{\frac{\Sigma f x^2}{N}}}$$

N = Total of frequencies.

<u>Example</u> :— Calculate the standard Deviation of the following Series.

Marks more than	0	10	20	30	40	50	60	70
No. of students	100	90	75	50	25	15	5	0

Class C.I	Midvalue M	f	fx	dx 31	fdx	fdx^2
0-10	5	10	50	-26	-260	6760
10-20	15	15	225	-16	-240	3840
20-30	25	25	625	-6	-150	900
30-40	35	25	875	+4	+100	400
40-50	45	10	450	+14	+140	1960
50-60	55	10	550	+24	+240	5760
60-70	65	5	325	+34	+170	5780
		N=100	Σfx 3100			Σfdx^2 =25400

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{3100}{100} = 31$$

$$\boxed{\bar{x} = 31}$$

$$\sigma = \sqrt{\frac{\Sigma fdx^2}{N}} = \sqrt{\frac{25400}{100}} = \sqrt{254} = 15.94$$

$$\boxed{\sigma = 15.94}$$

Short cut Method

C.I	(x) Midvalue	f	dx(35)	fdx	fdx^2
0-10	5	10	-30	-300	900
10-20	15	15	-20	-300	6000
20-30	25	25	-10	-250	2500
30-40	35	25	0	0	0
40-50	45	10	+10	+100	1000
50-60	55	10	+20	+200	4000
60-70	65	5	+30	+150	4500
		N=100		Σfdx -400	Σfdx^2 = 27000

$$\bar{x} = A + \frac{\Sigma fdx}{N} = 35 + \frac{-400}{100} = 35 - 4 = 31$$

$$\sigma = \frac{1}{N}\sqrt{\Sigma f dx^2 . N - (\Sigma f dx)^2}$$

$$= \frac{1}{100}\sqrt{27000 \times 100 - (-400)^2}$$

$$= \frac{1}{100}\sqrt{2700000 - 160000}$$

$$= \frac{1}{100}\sqrt{2540000}$$

$$= \frac{1}{100} \times 1594 = \boxed{15.94}$$

Step Deviation Method :- It is calculated by the following formula :-

$$\sigma = \frac{i}{N}\sqrt{\Sigma f dx^2 . N - (\Sigma f dx)^2}$$

i = common Factor.

Example :- Find out Standard Deviation from the following data :-

Marks :-	0-10	10-20	20-30	30-40	40-50
No. of students	10	15	10	10	5

Sol.

Marks	M.V x	f	dx (25)/10	fdx	fdx²
0-10	5	10	-2	-20	40
10-20	15	15	-1	-15	15
20-30	25	10	0	0	0
30-40	35	10	+1	+10	10
40-50	45	5	+2	+10	20
		N=50		Σfdx = -15	Σfdx² 85

$$\sigma = \frac{i}{N}\sqrt{\Sigma f dx^2 - N - (\Sigma f dx)^2}$$

$$= \frac{10}{50}\sqrt{85 \times 50 - (-15)^2}$$

$$= \frac{10}{50}\sqrt{4250 - 225}$$

$$= \frac{1}{5}\sqrt{4025}$$

$$= \frac{1}{5} \times 63.44 = 12.69$$

$$\boxed{\sigma = 12.69}$$

Varience & its Significance :-

The term Variance is used to describe the square of the Standard deviation and was coined by R.A Fisher in 1913. It is significant because of the the fact that it is a capable of a very exhaustive type of quantitative analysis. When a statistician has to deal with a phenomenon that is affected by a number of Variables, the analysis of Variance is used so that the effects of various factors can be studied in isolation. It is calculated by the formula

$$\text{Variance} = \frac{(X - \bar{X})^2}{N} \text{ or } \frac{\Sigma dx^2}{N}.$$

$$= \sigma^2$$

where assumed mean is taken and step deviation are calculated,

$$\text{Variance} = \left[\frac{\Sigma f dx^2}{N} - \left(\frac{\Sigma f dx}{N}\right)^2\right] \times i^2$$

Example :- Calculate Variance for the following series.

No. of Persons	4	6	8	10	4	6
Class Interval	0-10	10-20	20-30	30-40	40-50	50-60

Soln

Class Interval C.I	No. of Persons f	Mid Value M	$dx = \frac{M-A}{i}$	fdx	fdx²
0-10	4	5	5-25/5 = -4	-16	256
10-20	6	15	15-25/5 = -2	-12	144
20-30	8	25	25-25/5 = 0	0	0
30-40	10	35	35-25/5 = 2	+20	400
40-50	4	45	45-25/5 = 4	+16	256
50-60	6	55	55-25/5 = 6	+36	1296
	N=38			Σfdx = 44	Σfdx² = 2208

Let A = 25

$$\text{Variance} = \left[\frac{\Sigma f dx^2}{N} - \left(\frac{\Sigma f dx}{N}\right)^2\right] \times i^2$$

= ~~$\frac{44}{38}$~~ -

$$= \left[\frac{2208}{38} - \left(\frac{44}{38}\right)^2\right] \times 5^2$$

= 56.76 × 25

Variance = 1419 Ans.

Co-efficient of Variation: — The measure of co-efficient of Variation was introduced by Karl Pearson in 1895. It indicates the relation between the standard deviation (σ) and arithmetic mean expressed, in the percentage terms. This measure is used to compare variability homogeneity stability, uniformity and consistency in two sets of data. The series having higher co-efficient of variation has higher degree of variability.

It is calculated by the following formula:-

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

where,
C.V = Co-efficient of Variation.
σ = Standard Deviation.
$\bar{x}$ = Arithmatic Mean.

Example:- The airthmatic mean of the runs secured by three batsmen, Vijay, Subhash and Kumar. In the same series of 10 innings are 50, 48, and 12 respectively. The standard deviation of their runs are, 15, 12 & 2 respectively. Who is the most consistent of the three.

Sol.

$$C.V. \text{ of (Vijay)} = \frac{\sigma}{x} \times 100 = \frac{15}{50} \times 100 = 30\%$$

$$C.V. \text{ of (Subash)} = \frac{\sigma}{x} \times 100 = \frac{12}{48} \times 100 = 25\%$$

$$C.V. \text{ of (Kumar)} = \frac{\sigma}{x} \times 100 = \frac{2}{12} \times 100 = 16.67\%$$

∴ Out of three batsmen Kumar is more constant.

example :- A group of 100 selected students is having average 168.8 cm in height with a co-efficient of variation of 3.2, what is the standard deviation of their heights

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

$$3.2 = \frac{\sigma}{168.8} \times 100$$

$$3.2 \times 168.8 = 100\sigma$$

$$= 540.16 = 100\sigma$$

$$100\sigma = 540.16$$

$$= \sigma = \frac{540.16}{100}$$

$$\sigma = 5.4016 \text{ or } 5.4$$

$$\boxed{\sigma = 5.4}$$ Ans